"You Do It" Solution

Lecture 4: Elementary Probability

Prepared as a classroom-ready detailed solution set for ECOM5005

Exercise 1: Independent Stock-Screen Criteria

Question

Alex has developed a stock screen and uses the S&P 500 as his investment universe. He assumes that five screening criteria are **independent**. The fractions of S&P 500 stocks meeting criteria A–E are:

$$0.45, \quad 0.57, \quad 0.50, \quad 0.25, \quad 0.67.$$

Assuming the S&P 500 contains 500 stocks, how many stocks will pass **all five** criteria?

A. 10

B. 11

C. 12

Detailed Solution

Step 1: Identify the probability concept.
The lecture states that, for **independent events**, the probability that all events occur is the product of their individual probabilities:

$$P(A \cap B) = P(A)P(B).$$

The same principle extends to five independent criteria:

$$P(A \cap B \cap C \cap D \cap E) = P(A)P(B)P(C)P(D)P(E).$$

Step 2: Substitute the screening probabilities.

$$\begin{aligned} P(\text{passes all five}) &= (0.45)(0.57)(0.50)(0.25)(0.67) \\ &= 0.021481875. \end{aligned}$$

Thus, approximately **2.148%** of the S&P 500 is expected to satisfy every criterion simultaneously.
Step 3: Convert the probability into an expected number of stocks.

$$500(0.021481875) = 10.7409375.$$

The calculation produces an **expected count** of about 10.74 stocks.
Step 4: Select the most appropriate whole-number answer.
A stock count must be a whole number. Among the listed choices, 10.74 is closest to 11. Therefore,

$$\boxed{\text{B. 11 stocks (approximately)}}.$$

Exercise 1: Interpretation and Final Answer

Why We Multiply

The five screening criteria are explicitly assumed to be **independent**. This means that satisfying one criterion does not change the probability of satisfying another. Under this assumption, the probability of satisfying every criterion is found by multiplying the five marginal probabilities.

Probability versus Expected Count

The product

$$0.45 \times 0.57 \times 0.50 \times 0.25 \times 0.67 = 0.021481875$$

is a **probability**, not a number of stocks. Multiplying by 500 converts that probability into the expected number of stocks:

$$500 \times 0.021481875 = 10.7409375.$$

Because this is an expectation, it can be fractional even though the realised number of stocks must be an integer.

Interpretation

Only about **2.15%** of the investment universe is expected to pass all five screens simultaneously. Applying several independent filters therefore reduces the candidate set very quickly, even when each individual criterion is passed by a substantial fraction of stocks.

Answer Summary

Final answer:

$$\boxed{\text{Approximately 11 stocks; option B.}}$$

Calculation:

$$500(0.45)(0.57)(0.50)(0.25)(0.67) = 10.7409375 \approx 11.$$

Final Check

For independent screening criteria, **multiply** the individual probabilities first, then multiply by the size of the investment universe. Do not add the criterion probabilities, because the question asks for stocks satisfying *all* criteria, not at least one criterion.

Exercise 2: Probability of At Least One University Call

Question

You have applied for a Masters degree program at Curtin University and the University of Queensland (UQ). The probability of receiving a call from Curtin is 0.70 and from UQ is 0.50. The slide states that the events are **mutually exclusive and exhaustive**. What is the probability of receiving a call from at least one university?

A. 0.20
B. 0.85
C. 0.35

Detailed Solution

Step 1: Translate "at least one" into probability notation.
Let

$$C = \text{receive a call from Curtin}, \qquad U = \text{receive a call from UQ}.$$

The requested probability is

$$P(C \cup U).$$

The general addition rule is

$$P(C \cup U) = P(C) + P(U) - P(C \cap U).$$

Step 2: Check the assumptions written on the slide.
If the events are **mutually exclusive**, then

$$P(C \cap U) = 0,$$

so the addition rule would give

$$P(C \cup U) = 0.70 + 0.50 = 1.20.$$

But a probability cannot exceed 1. Therefore, the given probabilities cannot be mutually exclusive. If the events are also **collectively exhaustive**, then at least one event must occur, implying

$$P(C \cup U) = 1.$$

This also conflicts with the calculation above. Hence the assumptions on the slide are internally inconsistent with $P(C) = 0.70$ and $P(U) = 0.50$.
Step 3: Compare with the multiple-choice options.
None of the three listed options is correct under the statement exactly as written. Therefore, the mathematically correct conclusion is:

$$\boxed{\text{The question is inconsistent as written.}}$$

Exercise 2: Likely Intended Interpretation and Final Answer

Likely Intended Assumption

The available answer choices strongly indicate that the intended assumption was that the two call events are **independent**, rather than mutually exclusive and exhaustive. Under independence,

$$P(C \cap U) = P(C)P(U) = (0.70)(0.50) = 0.35.$$

after which the general addition rule gives

$$\begin{aligned} P(C \cup U) &= P(C) + P(U) - P(C \cap U) \\ &= 0.70 + 0.50 - 0.35 \\ &= 0.85. \end{aligned}$$

Thus, under the likely intended interpretation,

$$\boxed{\text{B. } 0.85}.$$

Interpretation

A probability of 0.85 means there is an **85% chance of receiving at least one call** from the two universities, assuming the two call events are independent. This includes three possibilities: Curtin only, UQ only, or both universities.

Why We Subtract the Overlap

Adding $0.70 + 0.50$ counts the outcome "both universities call" twice. Under independence, that overlap has probability

$$P(C \cap U) = 0.35.$$

Subtracting it once avoids double counting:

$$0.70 + 0.50 - 0.35 = 0.85.$$

Answer Summary

As the slide is written: the assumptions are contradictory, so no listed option is mathematically valid.

Under the likely intended independent-events assumption:

$$\boxed{P(C \cup U) = 0.85, \text{ option B.}}$$

Final Check

Remember the distinction:

- **Mutually exclusive**: $P(C \cap U) = 0$.
- **Independent**: $P(C \cap U) = P(C)P(U)$.
- **Collectively exhaustive**: $P(C \cup U) = 1$.

These concepts are not interchangeable. With probabilities 0.70 and 0.50, the slide cannot consistently describe the events as both mutually exclusive and exhaustive.